

6.7 Financial Models

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate r , expressed as a decimal, compounded n times per year is

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Common values of n :

Compounded	annually	semi-annually	quarterly	monthly	daily
n	1	2	4	12	365

Example – Future Value of Money

Find how much you would have in an account after 5 years if you invest \$500 at 4% compounded quarterly. ← $n=4$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$
$$A = 500 \left(1 + \frac{.04}{4}\right)^{4(5)}$$
$$A = 500(1 + .04/4)^{20}$$
$$A = \$610.10$$

Your Turn

You make a \$1500 furniture purchase with no payments for 12 months. What is your account balance if you make no payments for a year and the store charges 18% interest compounded daily?

$$t = 1 \text{ year}$$

$$n = 365$$

$$A = 1500 \left(1 + \frac{.18}{365}\right)^{(365(1))}$$

$$A = \$1795.75$$

Continuous Compounding → Continuous Growth Model

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is

$$A = Pe^{rt}$$

Example

Using Continuous Compounding

Find the amount that results from investing a principal of \$1000 at an annual rate of 10% compounded continuously for 3 years.

$$A = Pe^{rt}$$
$$A = 1000e^{.10(3)}$$
$$A = \$1349.86$$

Example – Present Value of Money

You want to save money for a trip to Europe 5 years from now. The trip will cost you \$5000. How much should you invest now in an account that pays 15.6% compounded monthly to reach your goal in 5 years?

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$
$$5000 = P \left(1 + .156/12 \right)^{12(5)}$$
$$5000 = P \frac{\cancel{(1 + .156/12)^{60}}}{(1 + .156/12)^{60}}$$

$$P = \$2303.58$$

Your Turn

A family wants to start a Florida Prepaid savings plan for college. They have 18 years to save at 13.4% average rate of return annually with a future goal of \$20,000. How much do they have to invest now to reach their goal?

$$20,000 = P(1 + .134/1)^{1(18)}$$
$$20,000 = P(1.134)^{18}$$

$$\frac{20,000}{(1.134)^{18}} = P$$

$$P = \$2079.67$$

Example

$$\frac{2P}{P} = \frac{P(1+r)^{10}}{P}$$

$$A = 2P$$

Rate of Interest Required to Double an Investment

What rate of interest compounded annually ^{← n=1} is needed in order to double an investment in 10 years? ~~15 years?~~

$$A = P(1 + \frac{r}{1})^{1(10)}$$

$$\left. \begin{array}{l} \text{let } P=1 \\ A=2 \\ t=10 \\ n=1 \end{array} \right\}$$

$$2 = (1+r)^{10}$$

$$\sqrt[10]{2} = \sqrt[10]{(1+r)^{10}}$$

take the 10th root of both sides

$$1.071773... = 1+r$$

$$.071773... = r$$

7.18% interest rate will double the investment in 10 years

Example

Time Required to Double or Triple an Investment

(a) How long will it take for an investment to double in value if it earns 5% compounded continuously?

(b) How long will it take to triple at this rate?

$$A = Pe^{rt}$$

$$2 = e^{.05t}$$

$$\frac{\ln 2}{.05} = \frac{.05t}{.05}$$

$$t \approx 13.86 \text{ years}$$

Financial Planning – Who wants to retire as a millionaire?

$$\left. \begin{array}{l} t = 45 \text{ years} \\ A = \$1,000,000 \\ n = 1 \\ r = 15\% \end{array} \right\}$$

$$1,000,000 = P(1 + .15)^{45}$$

$$\frac{1,000,000}{(1.15)^{45}} = P$$

$$P = \$1856.08$$